TOPOLOGY III MID-TERM EXAM

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Each question is of 6 marks adding up to a total of 30 marks.

1. Compute the homology groups $H_i(\mathbb{R}^2, \mathbb{Z}^2)$ for all i, where $\mathbb{Z}^2 \longrightarrow \mathbb{R}^2$ is the obvious inclusion. Use this to compute the homology groups of a torus.

2. Prove the 5-lemma : In the diagram

$$\begin{array}{ccccccccc}
A & \xrightarrow{i} & B & \xrightarrow{j} & C & \xrightarrow{k} & D & \xrightarrow{l} & E \\
\alpha\downarrow & & \beta\downarrow & & \gamma\downarrow & & \delta\downarrow & & \epsilon\downarrow \\
A' & \xrightarrow{i'} & B' & \xrightarrow{j'} & C' & \xrightarrow{k'} & D' & \xrightarrow{l'} & E'
\end{array}$$

If the horizontal rows are exact and the maps α, β, δ and ϵ are isomorphisms then so is γ.

3. Let

$$f : S^n \longrightarrow S^n$$

be a map of degree 0. Show that there exists points x and y such that $f(x) = x$ and $f(y) = -y$.

4. Let X be the quotient space of S^2 under the identification $x \sim -x$ for x in the equator S^1. Compute the homology groups $H_i(X)$.

5. Show that if X is a CW-complex the the group $H_n(X^n)$ of the n-skeleton X^n is free. Hint: Show that it is a subgroup of a free abelian group.